Stage A: Concrete apparatus

Stage of Development:

Year One

- Subtract numbers up to 20 (including zero)

Year Two

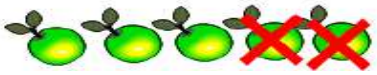
- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and units
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers

Underlying skills:

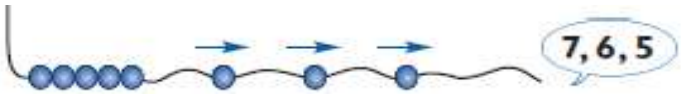
- Number recognition 0 -10 then 20
- Know that numbers identify how many objects are in a set
- Recognise numbers and represent them using objects
- Count objects accurately using one to one correspondence matching a number name to each object
- Being able to count backwards from numbers up to 10 then 20
- Being able to count on from numbers other than 0

<u>Taking away</u>

5 – 2 = 3

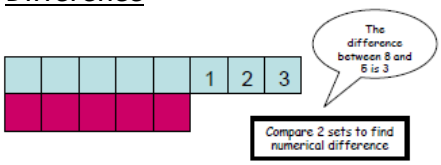


8 – 3 = 5

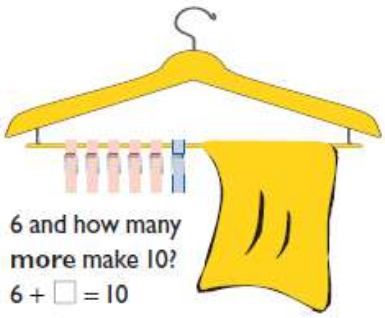


10 – 4 = 6

<u>Difference</u>



How many more?
Which has more? Less? Greater? Fewer? What's the difference?



Stage B: **Number lines**

Stage of Development:

Year One

- Subtract numbers up to 20 (including zero)

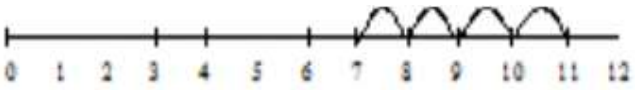
Year Two

- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and units
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
  -

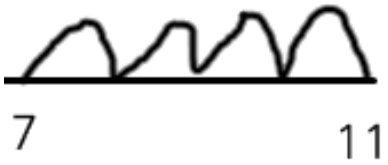
Underlying skills:

- Number recognition 0 -20 then to 100
- Counting back from any number up to 20 then to 100
- Interpreting a number sentence – knowing what number to start at and how many jumps to do

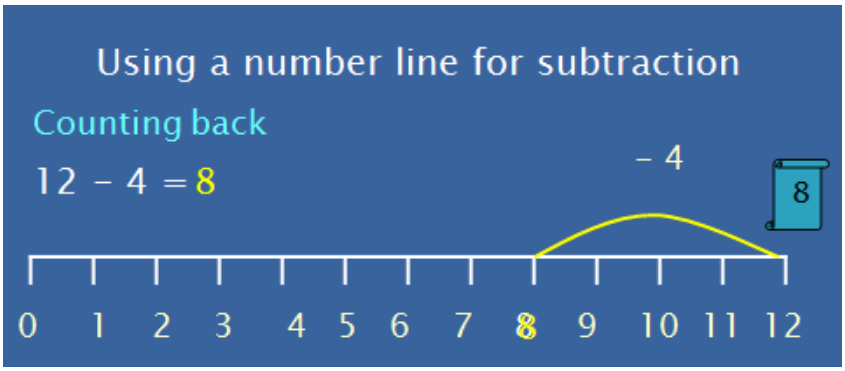
1) Counting back on a numbered number line



2) Counting back on an empty number line



3) Increased efficiency using known facts



Stage C: **Written at KS1 and mental at KS2**
Number lines – Complementary addition finding a small difference (less than 10)

Stage of Development:

Year One

- Subtract numbers up to 20 (including zero)

Year Two

- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and units
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers

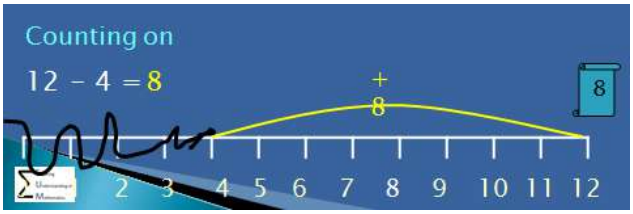
Underlying skills:

- Knowing to start at the lesser number and count on
- Understanding difference as how many more and that the number of jumps is the answer
- Being able to count on from any number up to 20

1) Using numbered number lines

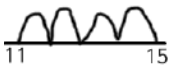
15 – 11 = 4 11 + __ = 15

The following approach helps pupils to make the connection between find the difference and take away. This must be used when first introducing subtraction by counting forwards (complimentary addition) as it makes the link between finding the difference and taking away.
For example to find the difference between 4 and 12 find the 12 on the number line, then 'take away the 4' (scribble 4 out on the number line to make the connection). Pupils can then understand the difference is what remains



2) Using an empty number line(counting forwards)

15 – 11 = 4 11 + __ = 15



Stage D: Number lines – Complementary addition for subtraction bridging through multiples of 10

Stage of Development:

Year One

- Subtract numbers up to 20 (including zero)

Year Two

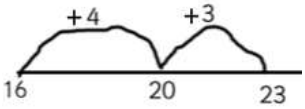
- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and units
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers

Underlying skills:

- Number bonds to 10 and 100
- Know the next multiple of 10 and the associated number bond
- Understanding the place value of numbers
- Adding multiples of 10 and 100
- Adding 2 and 3 digit numbers
- Number bonds in decimals for KS2 mental method

1) Differences under 10

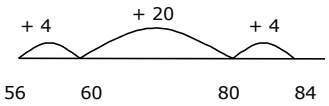
23 – 16 = 7 16 + ___ = 23



Bead strings can be used to demonstrate bridging through 10

2) Larger differences

84 – 56 = 28 56 + ___ = 84



Initially the jump of 20 may need to be broken down to 2 jumps of 10

Stage F: Formal written method for subtraction: Decomposition
When presenting calculations vertically pupils are taught to start with the least significant digit.

Stage of development:

Year Three:

- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Year Four:

- Subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Year Five:

- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Year Six:

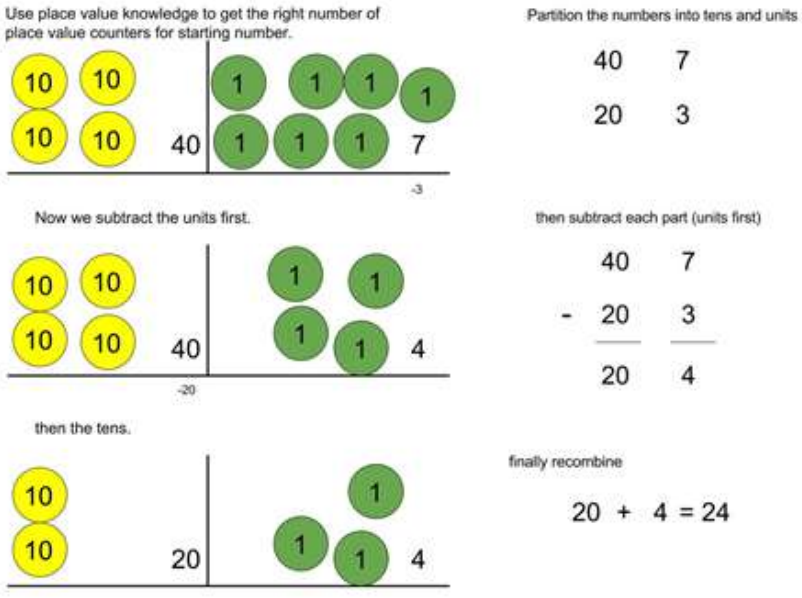
- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Underlying skills:

- Understanding the place value of each digit in a number
- Partitioning in different ways eg 43 = 40 + 3; 43 = 30 + 13
- Understanding the exchange/ regroup process – e.g. that 1 hundred and 3 tens is the same as 13 tens
- Fluency in all subtraction facts for numbers to 10

Step 1 Expanded

47-23=



Partition the numbers into tens and units

40 7

20 3

then subtract each part (units first)

40 7

\- 20 3

20 4

finally recombine

20 + 4 = 24

We do not use the models to represent 23 because it does not exist separately from the 47. It is **part of the 47**

This can, and should, be represented using a range of resources, including cubes, place value counters, Dienes cubes and other base 10 number rods.

Before moving on to the next stage it is important that the children have experience of partitioning numbers in a variety of different ways:

358

Could become:

300 50 8

or:

300 40 18

or:

300 30 28

or:

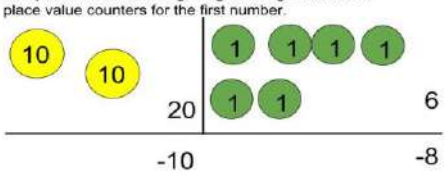
200 150 8

or:

200 140 18

26-18=

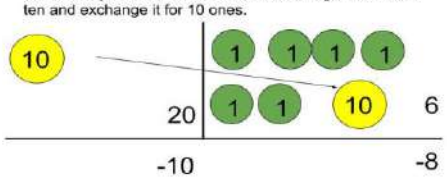
Use place value knowledge to get the right number of place value counters for the first number.



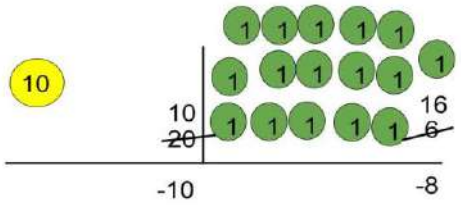
Partition the two numbers and write one under the other. At this stage we have partitioned both numbers into tens and units. This may not be the best way but it is a good place to start.

	20	6
−	10	8

As before we subtract the units first. This time we can't take 8 away from six so we need to exchange. We take a ten and exchange it for 10 ones.



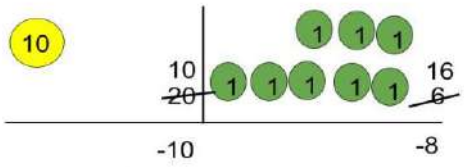
As before we subtract the units. However we can't take 8 from 6. At this point we need to repartition our first number in a more useful way. Instead of 20 and 6 we can have 10 and 16 (we know this from previous work on partitioning in different ways).



	10	16
	~~20~~	~~6~~
−	10	8

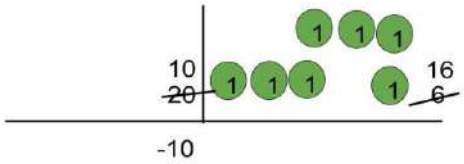
Now there is one ten and 16 ones. We can now take 8 from 16 so we do that.

From this point we can subtract 8 from 16. Then ten from ten



	10	16
	~~20~~	~~6~~
−	10	8
	0	8

Finally we take 1 ten away from one ten.



Finally we recombine them.

0 + 8 =8

The expanded method leads children to the more compact method so that they understand its structure and efficiency.

The amount of time that should be spent teaching and practising the expanded formal method will depend on how secure the children are in their recall of number facts and with partitioning and exchanging

Step 2 compact:
This is exactly the same process as the previous step but the jottings look a little different to increase efficiency. The repartitioning is done without fully partitioning out the numbers

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}3 \\ -\ 27 \\ \hline 16 \end{array}$$