

## Stage A: Grouping using concrete apparatus

### Stage of development

#### Year One

- Count in multiples of twos, fives and tens
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, arrays and pictorial representations with the support of the teacher.

#### Year Two

- Solve problems involving multiplication using materials and multiplication facts, including problems in contexts.

#### Underlying skills:

- Count objects accurately using one to one correspondence matching a number name to each object
- Number recognition 0 -10, then 20 and then 100
- Count up to 10 then 20 and then 100
- Understanding of the 'equal groups'

- Counting objects in groups e.g. socks, shoes, animal legs, coins, hands, fingers, toes etc

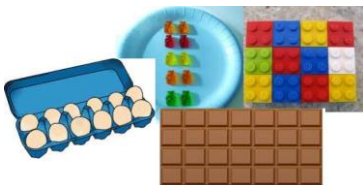


- Arrays

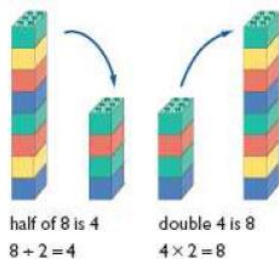
Practical objects; cubes, counting objects, pegs and pegboards etc.



Practical examples: Egg boxes, paint palettes, chocolate bars/boxes etc



- Halving and doubling



## Stage B: Multiplication as repeated addition

### Stage of development

#### Year One

- Connections between arrays, number patterns and counting in twos, fives and tens
- Solve one-step problems involving multiplication, by calculating the answer using arrays with the support of the teacher.

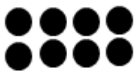
#### Year Two

- Calculate maths statements for multiplication within the multiplication tables and write them using the symbols  $\times$  and  $=$  (developing on from repeated addition)
- Understand and show that multiplication is commutative
- $3 \times 4 = 4 \times 3$

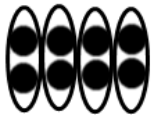
### Underlying skills:

- Understand that multiplication means equal groups of
- Count forwards in steps of 2, 5 and 10

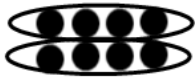
#### Arrays



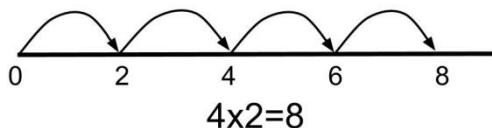
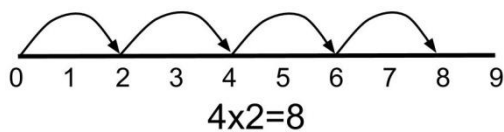
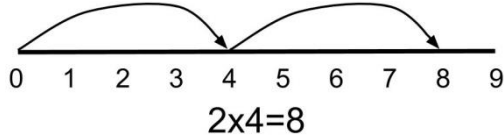
- $4 \times 2$     $2 + 2 + 2 + 2$



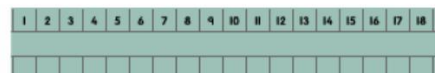
- $2 \times 4$     $4 + 4$



#### Number lines



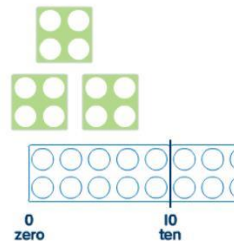
This can also be represented using Numicon:



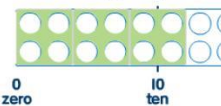
What is 3 groups of 4?



Fill the 4s into the number track and see where they fill to.



What is 3 groups of 4?

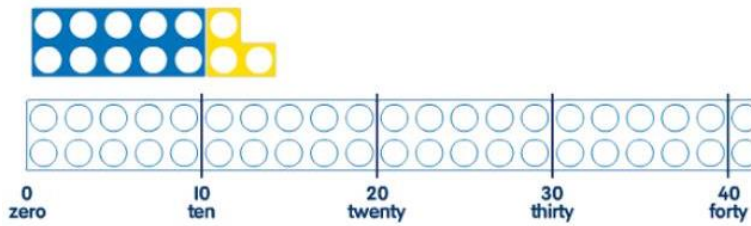


Fill the 4s into the 10s number line and see where they fill to.

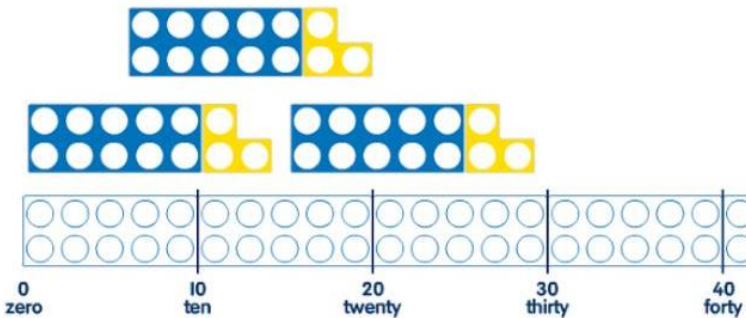
## Moving to 2 digit numbers and partitioning

 $3 \times 13$ 

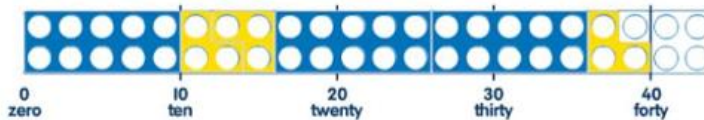
First we make 13 using the numicon. There is no 13 shape so we need to combine two other shapes. There are a number of ways of doing this but the easiest for us is to use a ten shape if we can.



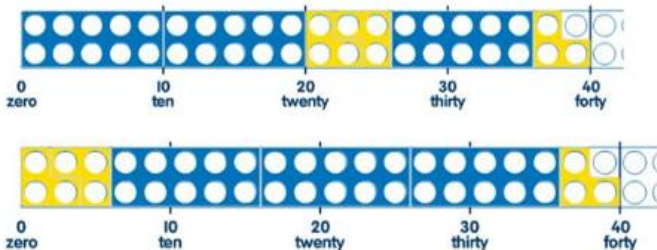
Now we need 3 of these



As before we place all three onto the numberline, because we have an odd number we have had to rotate the second block of 13 to fit properly.



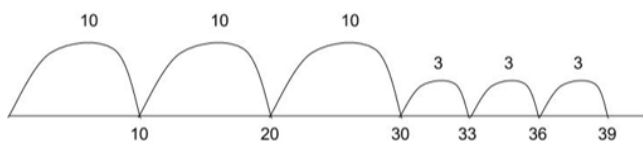
Note that we can place the shapes in any order, so long as we move the three shapes to fill in any gaps



One way of organising them is to put the ten shapes first then put the three shapes next like this:



This means we have 3 ten shapes and 3 three shapes ( $3 \times 10$  and  $3 \times 3$ ). We can then start to represent this on a numberline:



Or possibly:



## Stage C: Grid Method

**THIS IS THE INFORMAL METHOD AND PUPILS NEED TO BE TAUGHT THE FORMAL METHODS IN STEPS 3 AND 4 IN STAGE D. THIS IS STILL A CRUCIAL STEP.**

## Stages of development

## Year Three

- Multiply two digit numbers x one digit numbers using mental and progressing to formal written methods
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

## Year Four

- Multiply two-digit and three-digit numbers by a one digit number

## Year Five

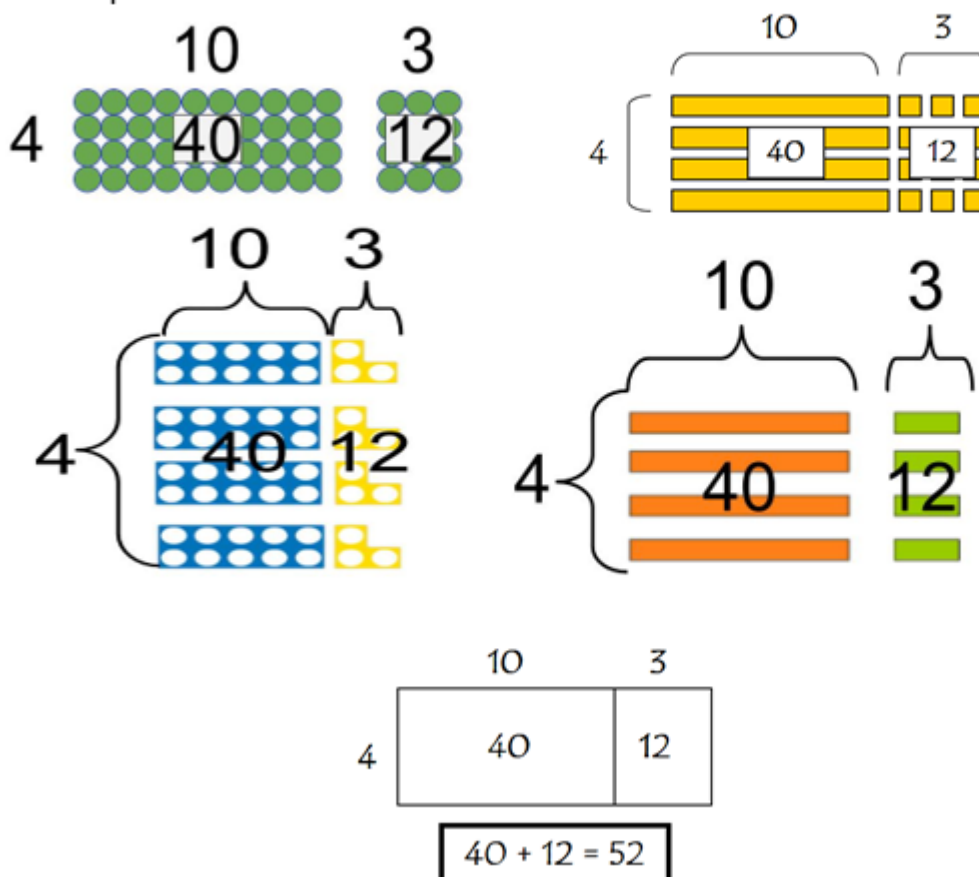
- Multiply numbers up to 4 digits by a one- or two-digit number

## Year Six

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number
- Multiply one-digit numbers with up to two decimal places by whole numbers

Underlying skills:

- Partition numbers appropriately
  - Recall appropriate multiplication facts
  - Multiply and divide numbers by 10/100/1000
  - Add together 2, 3 or 4 digit numbers, including decimals
  - Multiply multiples of 10
  - Multiply decimals, understanding the place value
- Representation moving to written method. The representation should be done using a range of different resources some of which might include; cubes, Diennes, objects and Numicon.  $13 \times 4$  could be represented like this:



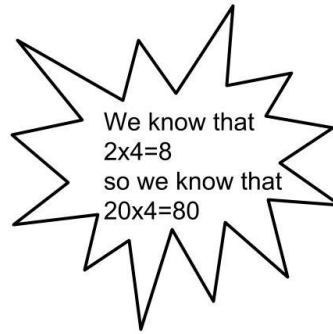
1 digit x 2 digit

For 2 digit numbers over 19 the mental calculation strategies from stage B will need to be thoroughly embedded.

$23 \times 4 =$

x	20	3
4	80	12

$$+ \begin{array}{r} 80 \\ 12 \\ \hline 92 \end{array}$$

1 digit x 3 digit

$4 \times 234 =$

x	200	30	4
4	800	120	16

$$+ \begin{array}{r} 800 \\ 120 \\ 16 \\ \hline 936 \end{array}$$

2 digit x 2 digit

$23 \times 34 =$

x	20	3
30	600	90
4	80	12

$$+ \begin{array}{r} 600 \\ 90 \\ 80 \\ 12 \\ \hline 782 \end{array}$$

1 digit x decimal

$4 \times 23.4 =$

x	20	3	0.4
4	80	12	1.6

$$+ \begin{array}{r} 80 \\ 12 \\ 1.6 \\ \hline 93.6 \end{array}$$

Mental strategies:

Place value knowledge to understand and explain:

If  $3 \times 6 = 18$  then:

$30 \times 6 = 180$

$3 \times 60 = 180$

$300 \times 6 = 1800$

$3 \times 600 = 1800$

$3000 \times 6 = 18000$

$3 \times 6000 = 18000$

**Stage D: FORMAL METHODS****Stage of development:****Year Five**

- Multiply numbers up to 4 digits by a one-digit or two-digit number using a formal method

**Year Six**

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

**Underlying skills**

- Use facts up to  $12 \times 12$  to derive facts involving multiples 10/1000
- Partition numbers appropriately
- Recall appropriate multiplication facts
- Multiply numbers by 10, 100, 1000
- Multiply multiples of 10
- Add together 2, 3 or 4 digit numbers

Familiarity with the grid method and the reasons for each step are essential before this stage can be taught with understanding

<b>X</b>	<b>20</b>	<b>4</b>	
<b>10</b>	200	40	
<b>6</b>	120	24	

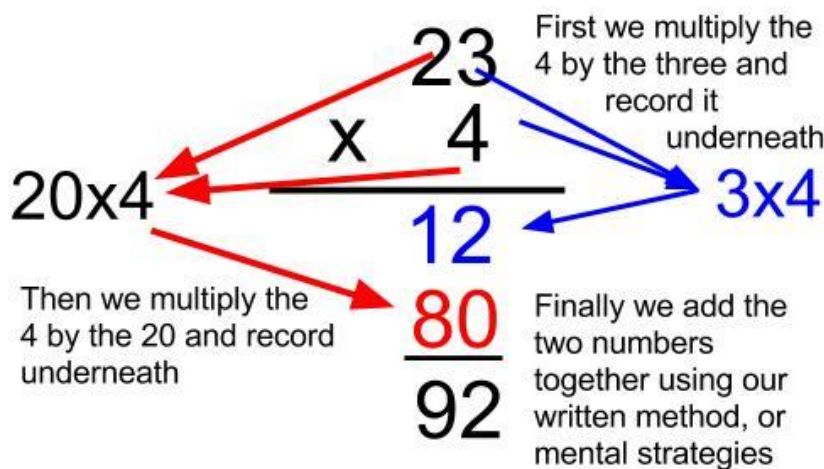
<b>240</b>
<b>+ 144</b>

<b>= 384</b>
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This format of adding back together should be shown to the children to support their understanding. Note the column for adding at the end is the same as those in the long method.

There are a number of steps to work through and children should demonstrate a good understanding of each as well as confidence with the place value before they move on to the next step towards the formal written methods.

**Step 1: 2 digit x 1 digit (expanded)**



**Step 2: 2 digit x 2 digit (expanded)**

$$\begin{array}{r}
 23 \times 34 = \\
 \begin{array}{r}
 20 \times 4 \\
 20 \times 30 \\
 + 3 \times 4 \\
 + 3 \times 30 \\
 \hline
 12 \\
 90 \\
 80 \\
 600 \\
 \hline
 782
 \end{array}
 \end{array}$$

1

In this method to start with we multiply the 3 by the 4 and record underneath. Then we multiply the 3 by the 30 and record this below the previous answer. Next we multiply the 20 by the 4. Finally we multiply the 20 by the 30.

We then add all the answers together using our written method or mental strategies

**Step 3: 2 digit x 1 digit (compact) Short Multiplication**

$$\begin{array}{r}
 23 \\
 \times 4 \\
 \hline
 92
 \end{array}$$

When we multiply the four by the 20 we then add the ten we carried earlier

As with the longer version the 4 is multiplied by the 3 but this time we record the 2 in the units and we carry the ten

**Step 4: 2 digit x 2 digit (compact) Long Multiplication**

$$\begin{array}{r}
 23 \times 34 = \\
 \begin{array}{r}
 20 \times 4 \\
 20 \times 30 \\
 + 3 \times 4 \\
 + 3 \times 30 \\
 \hline
 102 \\
 680 \\
 \hline
 782
 \end{array}
 \end{array}$$

In this version of the method we follow all the same steps as the previous version of the method except this time we record in a more compact way.

As before we start with multiplying the 3 by the 4 But this time we record the 2 in the units column and we carry the 10. Then we multiply the 3 by the 30 and add the ten we have carried.

Next we multiply the 20 by the 4 and because we know there are not going to be any units we can simply record that. Then we multiply the 20 by the 30 and place that in the columns next to the previous answer.

Finally we then add all the answers together using our written method or mental strategies