Stage A: Grouping using concrete apparatus

Stage of development

Year One

- Count in multiples of twos, fives and tens
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, arrays and pictorial representations with the support of the teacher.

Year Two

- Solve problems involving multiplication using materials and multiplication facts, including problems in contexts.

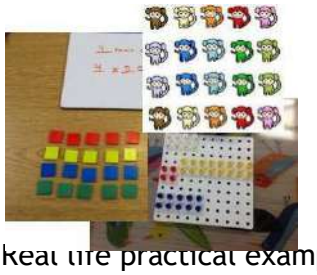
Underlying skills:

- Count objects accurately using one to one correspondence matching a number name to each object
- Number recognition 0 -10, then 20 and then 100
- Count up to 10 then 20 and then 100
- Understanding of the 'equal groups'

1) Counting objects in groups e.g. socks, shoes, animal legs, coins, hands, fingers, toes etc

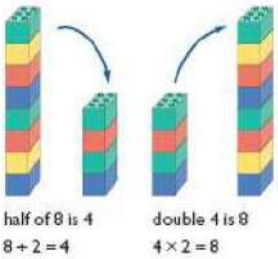
2..... 4..... 6.....

2) Arrays
Practical objects; cubes, counting objects, pegs and pegboards etc.



Real life practical examples: Egg boxes, paint palettes, chocolate bars/boxes etc

3) Halving and doubling



Stage B: Multiplication as repeated addition

Stage of development

Year One

- Connections between arrays, number patterns and counting in twos, fives and tens
- Solve one-step problems involving multiplication, by calculating the answer using arrays with the support of the teacher.

Year Two

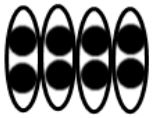
- Calculate maths statements for multiplication within the multiplication tables and write them using the symbols x and = (developing on from repeated addition)
- Understand and show that multiplication is commutative
- 3 x 4 = 4 x 3

Underlying skills:

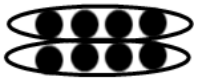
- Understand that multiplication means equal groups of
- Count forwards in steps of 2, 5 and 10

Arrays

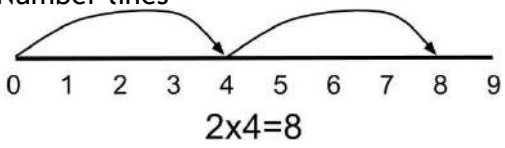
- 4 x 2 2 + 2 + 2 + 2

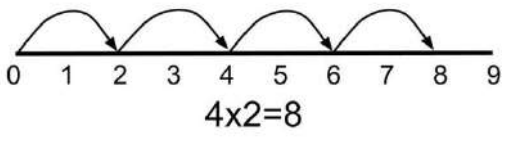


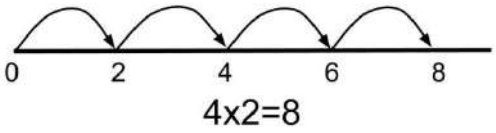
- 2 x 4 4 + 4



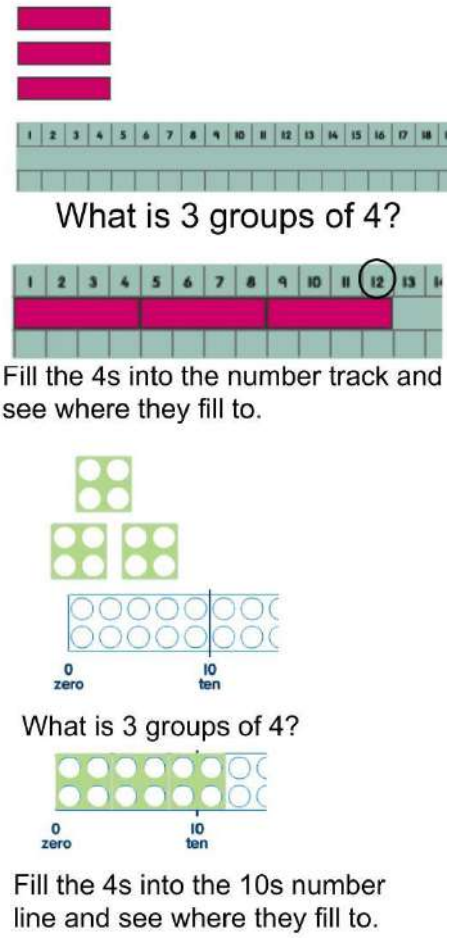
Number lines







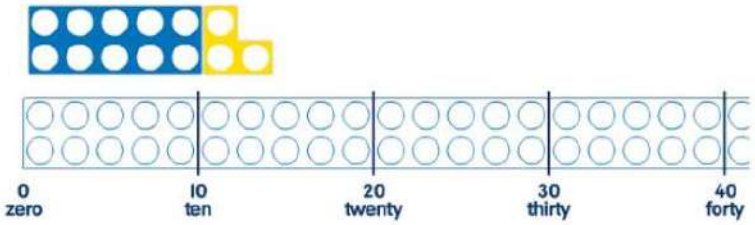
This can also be represented using Numicon:



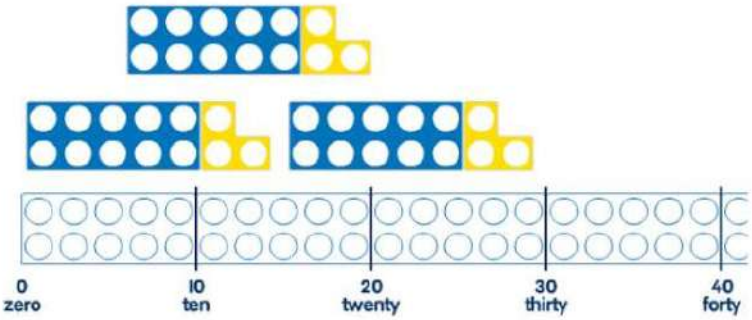
Moving to 2 digit numbers and partitioning

3x13

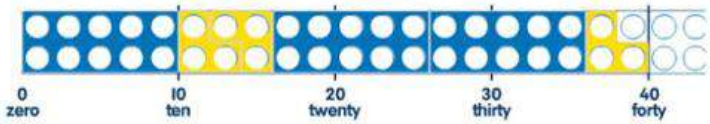
First we make 13 using the numicon. There is no 13 shape so we need to combine two other shapes.
There are a number of ways of doing this but the easiest for us is to use a ten shape if we can.



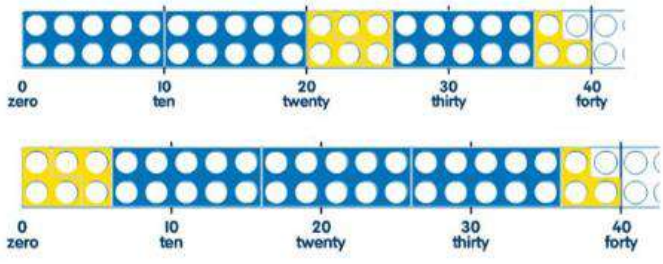
Now we need 3 of these



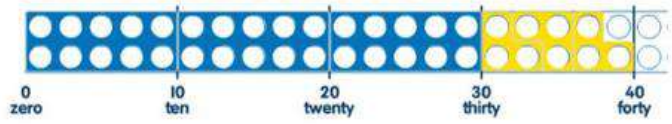
As before we place all three onto the numberline, because we have an odd number we have had to rotate the second block of 13 to fit properly.



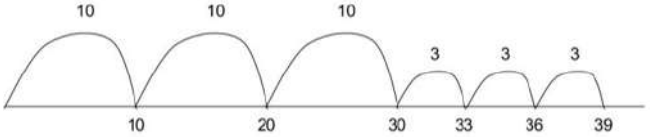
Note that we can place the shapes in any order, so long as we move the three shapes to fill in any gaps



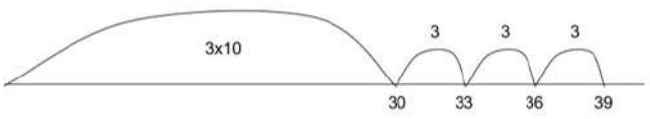
One way of organising them is to put the ten shapes first then put the three shapes next like this:



This means we have 3 ten shapes and 3 three shapes (3x10 and 3x3).
We can then start to represent this on a numberline:



Or possibly:



Stage C: Grid Method

THIS IS THE INFORMAL METHOD AND PUPILS NEED TO BE TAUGHT THE FORMAL METHODS IN STEPS 3 AND 4 IN STAGE D. THIS IS STILL A CRUCIAL STEP.

Stages of development

Year Three

- Multiply two digit numbers x one digit numbers using mental and progressing to formal written methods
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Year Four

- Multiply two-digit and three-digit numbers by a one digit number

Year Five

- Multiply numbers up to 4 digits by a one- or two-digit number

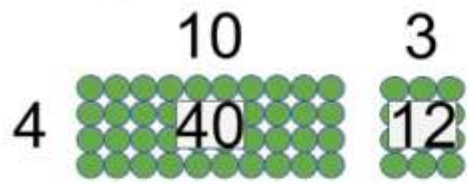
Year Six

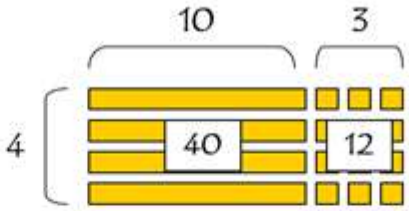
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number
- Multiply one-digit numbers with up to two decimal places by whole numbers

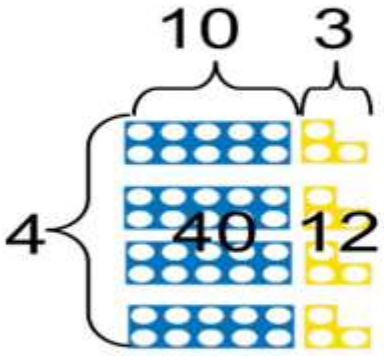
Underlying skills:

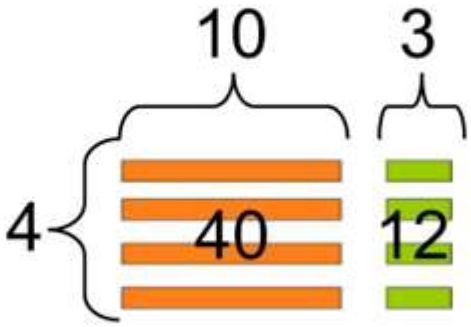
- Partition numbers appropriately
- Recall appropriate multiplication facts
- Multiply and divide numbers by 10/100/1000
- Add together 2, 3 or 4 digit numbers, including decimals
- Multiply multiples of 10
- Multiply decimals, understanding the place value

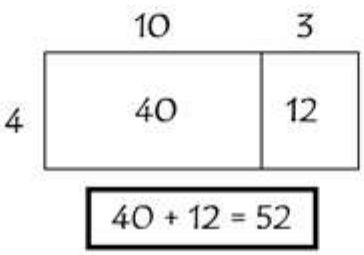
- Representation moving to written method. The representation should be done using a range of different resources some of which might include; cubes, Diennes, objects and Numicon. 13x4 could be represented like this:











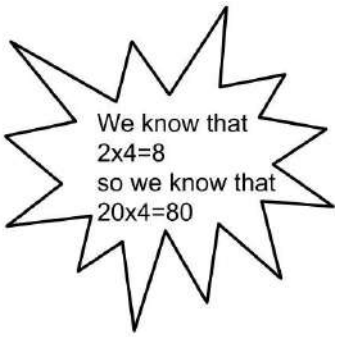
1 digit x 2 digit

For 2 digit numbers over 19 the mental calculation strategies from stage B will need to be thoroughly embedded.

23x4=

x	20	3
4	80	12

80
+ 12
92



1 digit x 3 digit

4x234=

x	200	30	4
4	800	120	16

800
120
+ 16
936

2 digit x 2 digit

23x34=

x	20	3
30	600	90
4	80	12

600
90
80
+ 12
782

1 digit x decimal

4x23.4=

x	20	3	0.4
4	80	12	1.6

80
12
+ 1.6
93.6

Mental strategies:

Place value knowledge to understand and explain:

If 3x6=18 then:

30x6=180	300x6=1800	3000x6=18000
3x60=180	3x600=1800	3x6000=18000

Stage D: FORMAL METHODS

Stage of development:

Year Five

- Multiply numbers up to 4 digits by a one-digit or two-digit number using a formal method

Year Six

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Underlying skills

- Use facts up to 12x12 to derive facts involving multiples 10/1000
- Partition numbers appropriately
- Recall appropriate multiplication facts
- Multiply numbers by 10, 100, 1000
- Multiply multiples of 10
- Add together 2, 3 or 4 digit numbers

Familiarity with the grid method and the reasons for each step are essential before this stage can be taught with understanding

X	20	4
10	200	40
6	120	24

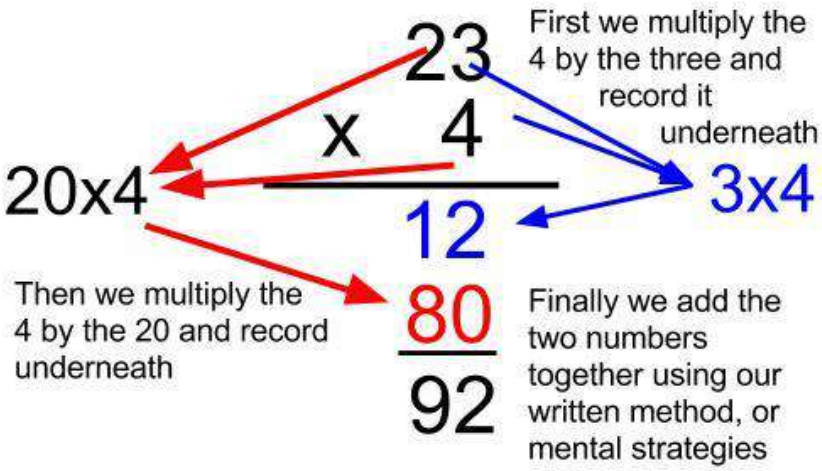
240
+ 144

= 384

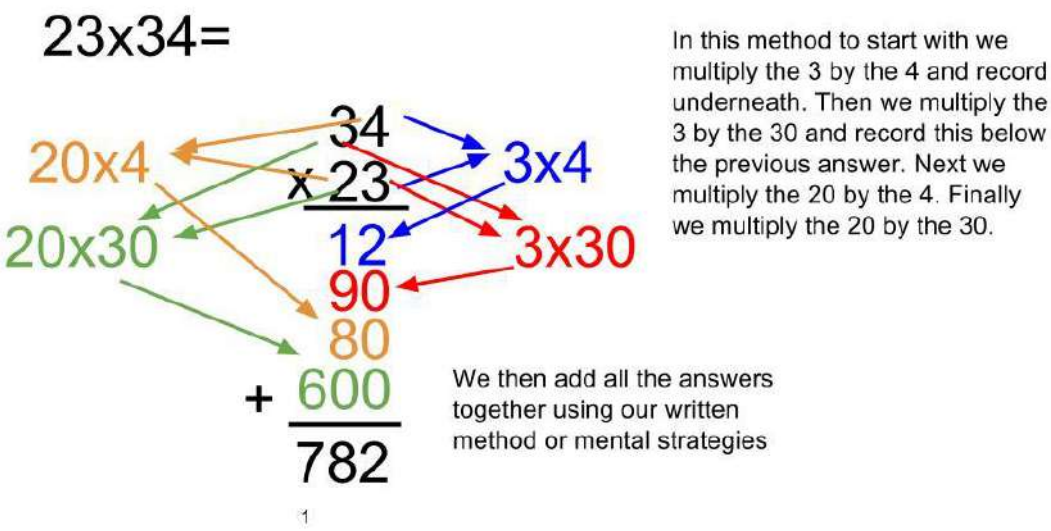
This format of adding back together should be shown to the children to support their understanding. Note the column for adding at the end is the same as those in the long method.

There are a number of steps to work through and children should demonstrate a good understanding of each as well as confidence with the place value before they move on to the next step towards the formal written methods.

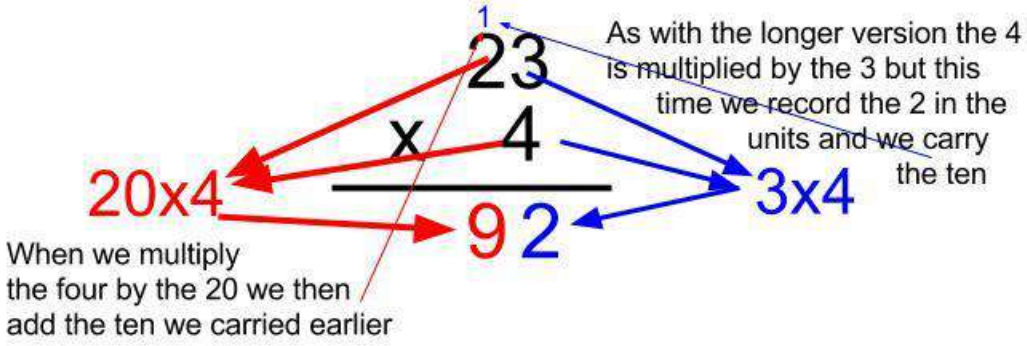
Step 1: 2 digit x 1 digit (expanded)



Step 2: 2 digit x 2 digit (expanded)



Step 3: 2 digit x 1 digit (compact) Short Multiplication



Step 4: 2 digit x 2 digit (compact) Long Multiplication