Stage A: Models of division

Stage of development

Year One

- Solve one step problems involving division by using concrete objects, pictorial representations and arrays (with support)

Year Two

- Solve problems involving division using materials, arrays, repeated subtraction, mental methods and division facts, including problems in context
- Calculate mathematical statements for division within the multiplication tables and write them using division (÷) and equals (=) signs

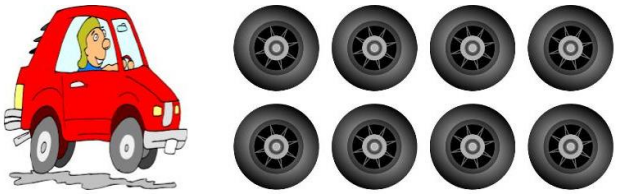
Underlying skills:

- One to one correspondence matching a number name to each object
- Understand sharing as everyone having the same
- The physical process of sharing "one for you , one for me"
- Knowing to count how many in a group to find the answer
- Creating equal groups
- Counting groups

- Share objects into equal groups and count how many in each in practical problem solving contexts

Language: 6 Sweets shared between 2 children. How many sweets each?
One for you, one for me.
Count how many each child has

- Group objects into groups of equal size



Language: Each car needs 4 wheels. If I have 8 wheels how many cars can I make?
How many groups of 4 can I get from 8?
Count how many groups there are.

Stage B: Arrays and grouping

Stage of development

Year One

- Solve one step problems involving division by using concrete objects, pictorial representations and arrays (with support)

Year Two

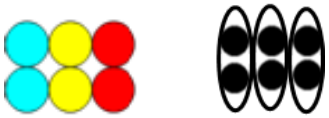
- Solve problem involving division using materials, arrays, repeated subtraction, mental methods and division facts, including problems in context
- calculate mathematical statements for division within the multiplication tables and write them using division (÷) and equals (=) signs

Underlying skills:

- Understand how to read division calculations as 'how many groups'
- Count forwards in steps of different single digit numbers accurately, see pitch
- Knowledge of multiplication tables

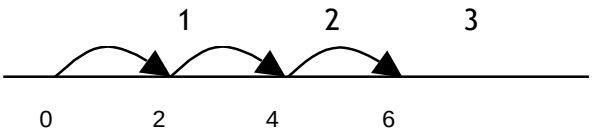
Understand division as 'how many groups' e.g 6 ÷ 2 = How many groups of 2 are there in 6?

- Arrays – count the groups



1 2 3

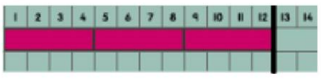
- Number line - jump in the appropriate size. Count the jumps.



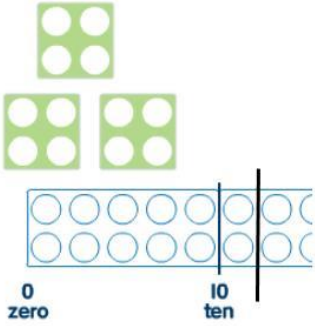
- Use Numicon to represent division as grouping, how many 4s can we fit in 12?

How many 4s can fit in 12?

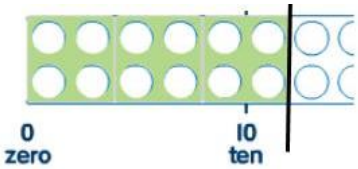
Start at 0 and add 4s until you can't fit any more



Count the number of 4s that fit in 12.

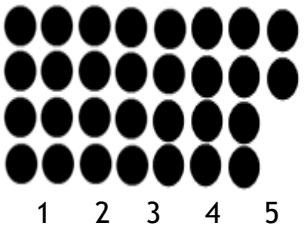


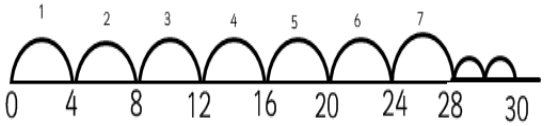
How many 4s fit in 12?



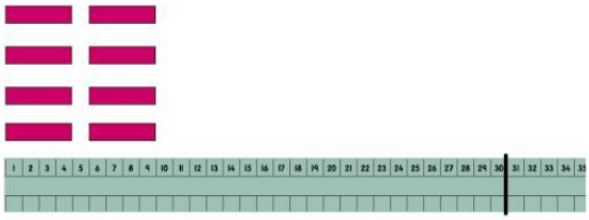
Fill the 4s into the 10s number until you can't fit any more and count the 4s.

- Include understanding of remainders
  30 ÷ 4 = 7r2

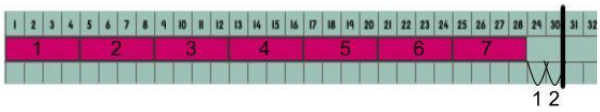




This can also be represented using Numicon number rods and track



How many 4s can fit in 30?



7 4s fit in 30 with two spaces not filled

This should be closely linked to multiplication as what is _ groups of _. If we know that 3x4= 12 then we know that 3 groups of 4 make 12 then how many groups of 4 are there in 12?

Stage C: grouping down

Stage of development

Year Two

- Solve problem involving division using materials, arrays, repeated subtraction, mental methods and division facts, including problems in context
- calculate mathematical statements for division within the multiplication tables and write them using division (÷) and equals (=) signs

Year Three

- Write and calculate mathematical statements for division, including for two digit numbers using mental and progressing to formal written methods

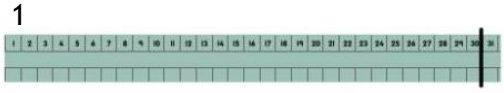
Underlying skills:

- Understand how to read number sentences as 'how many groups'
- Recall of appropriate multiplication facts
- Counting back in groups

Children need to be shown that finding how many groups can be done by starting at 0 and filling up or by starting at the top and working down. These should be done side by side so the children are clear that both methods give the same answer

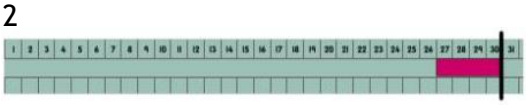
- Filling down on a number line.

1



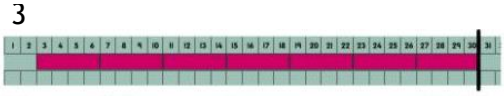
How many 4s fit in 30?

2



Start filling from the end.

3



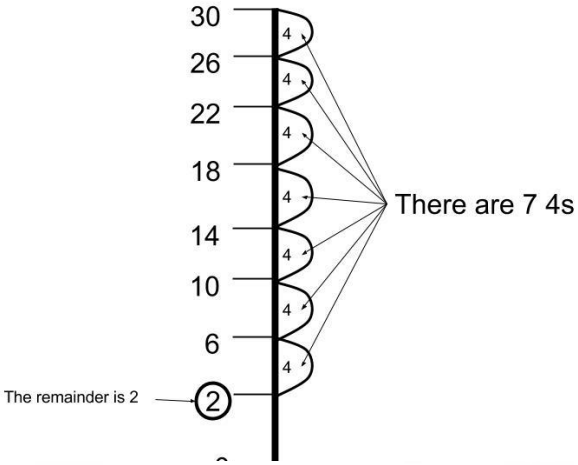
Keep filling until no more will fit.

4

look at the number to find the remainder.

The answer is still 7 remainder 2

- Represented on a vertical number line:

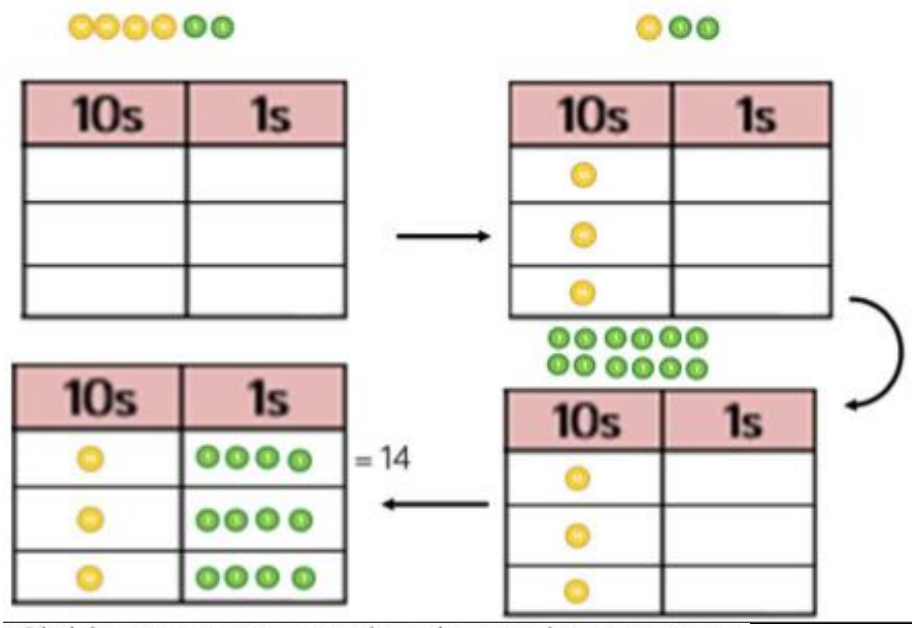


The answer is still 7 remainder 2

The numberline should be filling downwards in preparation for repeated subtraction

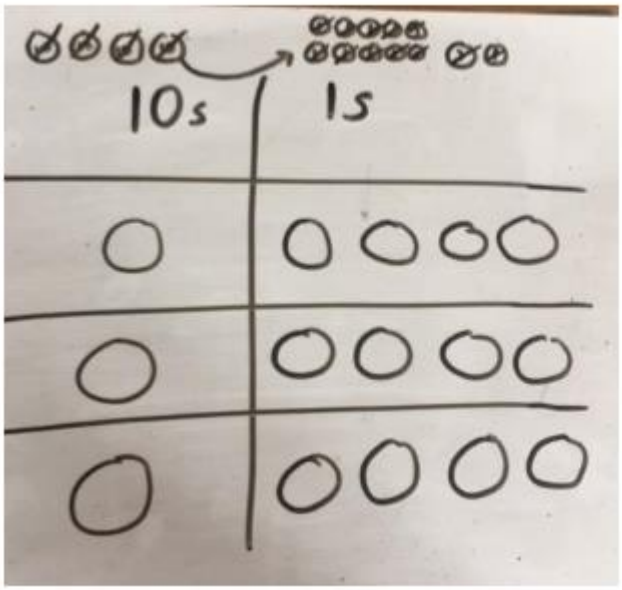
Stage C: Sharing with place value counters
Stage of development **Year Three** • Write and calculate mathematical statements for division, including for two digit numbers using mental and progressing to formal written methods **Year Four** • Divive two-digit and three-digit numbers by a one-digit number using formal written layout
Underlying skills: • Recall of appropriate multiplication facts • Understanding the exchange of a 10 for ten 1s

$42 \div 3 = 14$



Children to represent the place value counters pictorially.

Begin by sharing out the four 10s counters (42). In this calculation, three 10s counters can be shared into the three groups, with one 10 remaining. This 10 is exchanged to ten 1s. The ten 1s, along with the original 2 1s (from 42) are then shared between the three groups.



Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$
$42 = 30 + 12$
$30 \div 3 = 10$
$12 \div 3 = 4$
$10 + 4 = 14$

Stage E: Grouping

- Understand division as taking away groups at a time – different sized groups

72 ÷ 5 = 14r2

$$\begin{array}{r} 14\text{r}2 \\ 5\overline{)72} \\ -\ 50 \quad (10 \times 5) \\ \hline 22 \\ -\ 20 \quad (4 \times 5) \\ \hline 2 \end{array}$$

Fact box:
10 x 5 = 50
4 x 5 = 20

- Begin by establishing known possible groups by recalling multiplication facts for that number by writing out a fact box
- Use what you know to take away the biggest chunk possible
- Continue removing chunks until you can no longer do so. Any number left will be a remainder
- Add up the chunks you have taken.
- Some children may need to take smaller chunks until they develop efficiency

e.g. 196 ÷ 6

Early grouping

$$\begin{array}{r} 32\text{ r }4 \\ 6\overline{)196} \\ -\ 60 \quad (10 \times 6) \\ \hline 136 \\ -\ 60 \quad (10 \times 6) \\ \hline 76 \\ -\ 60 \quad (10 \times 6) \\ \hline 16 \\ -\ 12 \quad (2 \times 6) \\ \hline 4 \end{array}$$

Efficient grouping

$$\begin{array}{r} 32\text{ r }4 \\ 6\overline{)196} \\ -\ 180 \quad (30 \times 6) \\ \hline 16 \\ -\ 12 \quad (2 \times 6) \\ \hline 4 \end{array}$$

The remainder can also be written as a $32\frac{4}{6}$ ⟶ fraction $32\frac{2}{3}$

The remainder is written as the numerator on the top line and the number you have been dividing by goes underneath, the denominator.

- The subtraction aspects should ideally be done mentally. Where the child is incapable of this they should make use of the school approach with jottings next to their working out.
- Sensible choice of groups will help to avoid more difficult subtractions.

Year Four
Divide two-digit and three-digit numbers by a one-digit number using formal written layout

Year Five
Divide numbers up to 4 digits by a one-digit number using the formal written method of long division and interpret remainders appropriately for the context

Underlying skills:

- Recall of appropriate multiplication facts
- Using known number facts to help derive others using: place value
  e.g. 2 x 4 = 8 so 20 x 4 = 80
  doubling:
  10 x 6 = 60 so 20 x 6 = 120
  adding known facts
  10 x 3 = 30 5 x 3 = 15 so 15 x 3 = 45
- Mental subtraction
- Knowledge of school written subtraction method for more difficult calculations

New Vocabulary:

For fraction and decimal remainders:
fraction, decimal, equivalent, simplest form

For decomposition:
exchange, regroup

Stage F Long Division (Formal method)	
Place value can be used to help identify the most efficient groups to take (this is long division). $756 \div 8 =$ HTU $8 \overline{)756}$ We are looking for the best groups to take away each time, note not necessarily the biggest If we take each digit in order then we can see what the best groups of 8 to take are. What is the best number of 8s to take out of 700? If we use our place value knowledge then we know that we can't take any groups of 8 from 7 so we will move on (We can take groups of 8 from 700 but it's not the most useful groups). Note: the best group of 8 will be a multiple of 100 HTU $8 \overline{)756}$ Now we need to look at the first **two** digits. What is the best number of 8s to take from 750? Using our place value and times tables we know that we can get 9 8s in 75 because 9x8=72 so we can take a group of 90 8s (which will be 720) in 750 (we can take more but this is the best group to take). Note: the best group of 8 will be a multiple of 10 HTU 9 $8 \overline{)756}$ $-\ 720$ 36 We record this in the tens column as 9 tens because this is 90 groups of 7. To help with the link between long division and grouping we also record the groups here to start with. (90 x 7) Now we subtract the groups (because of the groups we have used the subtraction is fairly straightforward) HTU 9 $8 \overline{)756}$ $-\ 720$ 36 Now we need to see how many 8s there are in 36. We can get 4 8s in 36 (4x8=32) HTU 94 $8 \overline{)756}$ $-\ 720$ 36 $-\ 32$ 4 We record this in the units column as 4 because this is 4 groups of 8 To help with the link between long division and grouping we also record the groups here to start with. (4 x 8) Now we take those groups away	**Year Five** Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context **Year Six** Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
Underlying skills Calculation of larger multiples of the divisor Derivation of key facts using fact box Interpretation of the remainder as a decimal Interpretation of the remainder as a fraction Efficient subtraction (decomposition)	Vocabulary Remainder Quotient Divisor

Stage D: Short division (formal method)

Stage of Development

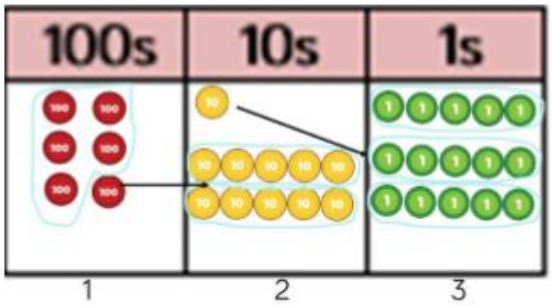
Year Four

- Divide two-digit and three-digit numbers by a one-digit number using formal written layout

Underlying skills

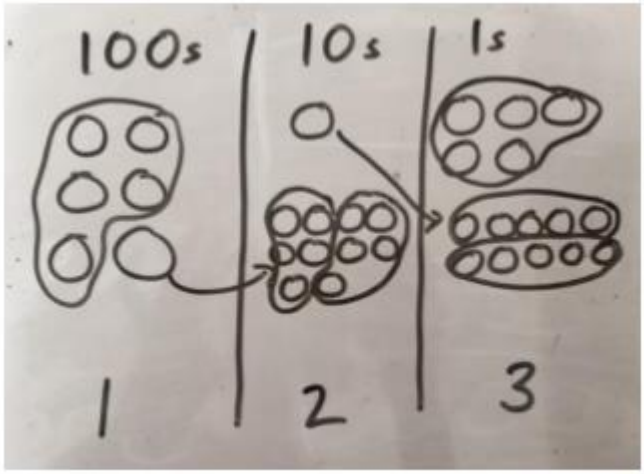
- Times tables and multiples
- Calculation of remainders
- Mental subtraction of multiples

615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

$$5\overline{)615}\quad 123$$

Stage E: Long Division

Stage of development

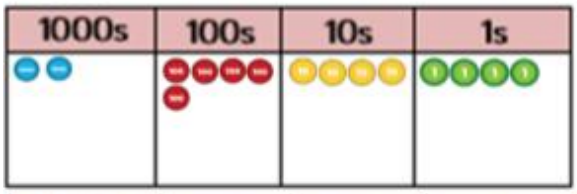
Year Five

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

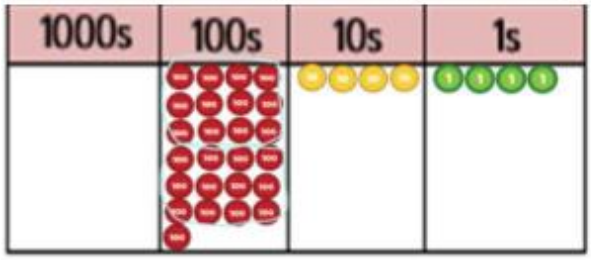
Year Six

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

2544 ÷ 12

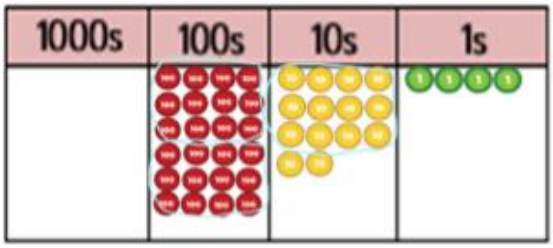


We can't group 2 thousands into groups of 12 so will exchange them.



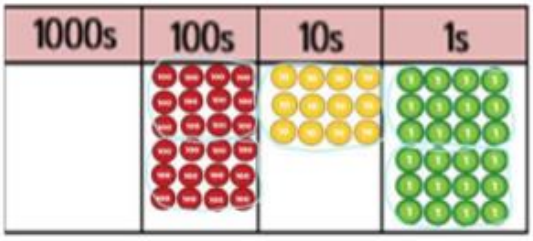
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$12\overline{)2544}\quad 02$$
$$24$$
$$1$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$