Stage A: Concrete apparatus

Stage of development :

Foundation stage
- Numbers up to 10

Year One
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Represent and use number bonds and related subtraction facts within 20

Year 2
- Use concrete objects and pictorial representations, including those involving numbers, quantities and measures
- Applying increasing knowledge of written methods

Underlying skills:
- Number recognition 0 -10 and then to 20
- Know that numbers identify how many objects are in a set
- Recognise numbers and represent them using objects
- Count objects accurately using one to one correspondence matching a number name to each object
- Know how to write each number
- Being able to count on from numbers other than 0

Understanding addition as combining two groups:

1) Relate addition to combining two groups of objects and counting them all.
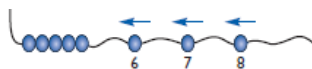

       3   +   2
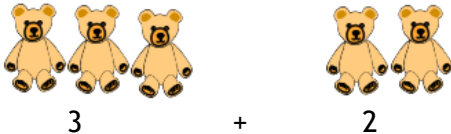
   Children will begin to record this pictorially   XXX   XX

2) Begin to relate addition to counting on using objects, bead strings, cubes and fingers.
   e.g.   5 + 3 =

   Children should develop a familiarity with the place value counters and know how the values of one directly relates to the value of the others

Stage B:	Number lines

Stage of development:

Year One
- Read, write and interpret mathematical statements involving addition (+) and equals (=) signs

- Add one-digit and two-digit numbers to 20, including zero

Year Two

- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
- A two-digit number and ones
- A two-digit number and tens
- Two two-digit numbers
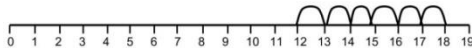- Adding three one-digit numbers

Underlying skills:
- Number bonds to 10 (or 20)
- Addition facts for all numbers up to and within 10 (20)
- Understanding that addition can be done in any order to aid calculation
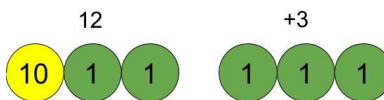- Counting on from any number up to 100 in ones and tens
-

Understanding addition as counting on:

1)   12+6=18

Start with the largest number and count on

Place value counters can be used to help develop fluency with counting on :

12   +3

10 1 1   1 1 1

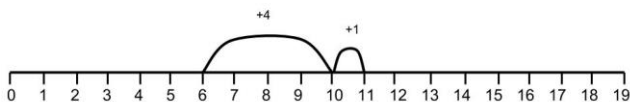0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Children will begin to record their work as number sentences.

2)   Using mental strategies to develop fluency and become quicker

6+5=

As the knowledge of number bonds develops children should be encouraged to see pairs they know and make jumps of more than one as they see them.
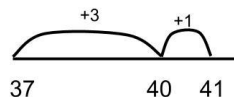
+4   +1

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

3)   Children will begin to draw their own empty number lines:

37+4

37   41

37+4

+3   +1

37   40   41

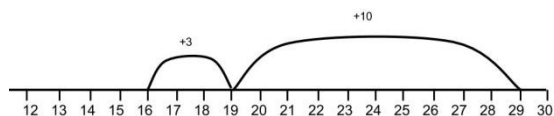**4)** Adding two 2-digit numbers

16+13=

Place value counters can also be used to develop fluency when adding two 2-digit numbers:

Children should be encouraged to add the units first then count on in tens.

5) This can then be done without the number line before progressing to a vertical method:
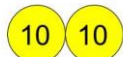
12+13

The numbers are partitioned and added separately before recombining at the end.

Units first

2 + 3= 5

10 + 10= 20

20 + 5= 25

Stage C: Start of the vertical (column) method

Stage of Development

Year Two
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
- A two-digit number and ones
- A two-digit number and tens
- Two two-digit numbers
- Adding three one-digit numbers

Year Three
- Add and subtract numbers with up to three digits

Underlying skills:
- Understanding the value of each digit in a number
- Being able to add multiples of 10
- Instant recall of addition facts for numbers up to 10
- Being able to add teen numbers to multiples of 10 mentally
- the value of each counter
- Know that the hundreds, tens and units should line up appropriately underneath each other

1) Start by partitioning the number into tens and units and add them separately. This can be represented using place value counters first and then written out in columns.

53+26=

Use place value knowledge to get the right number of place value counters for each number.
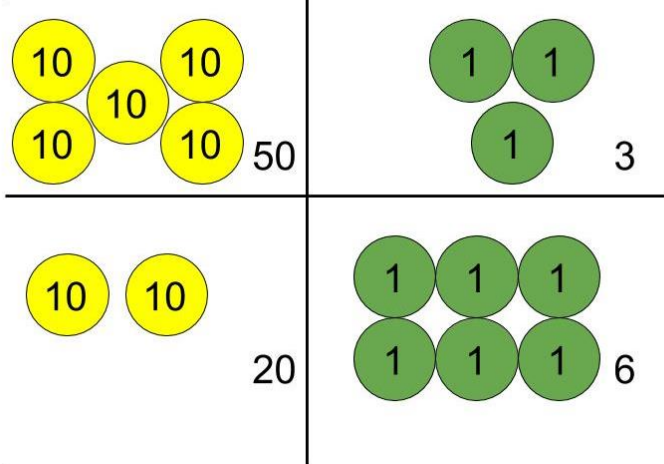
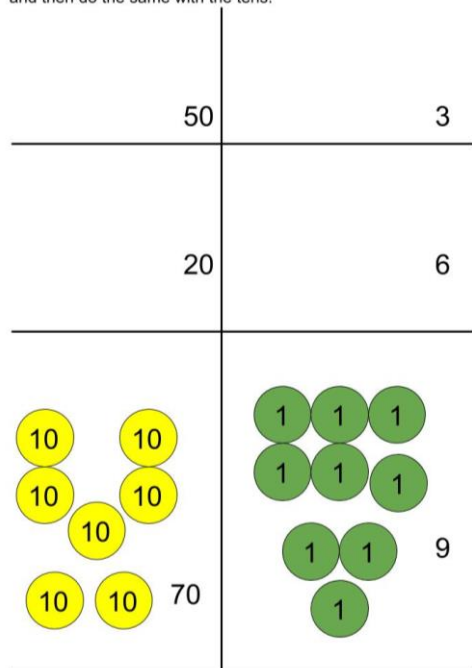53+26=

Partition the numbers into tens and units

50   3

20   6

Move the units together and count how many there are and then do the same with the tens.
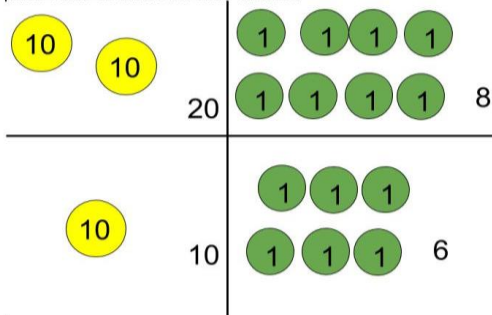
50	3
20	6

then add each part (units first)

$$\begin{array}{cc} 50 & 3 \\ +\ 20 & 6 \\ \hline 70 & 9 \end{array}$$
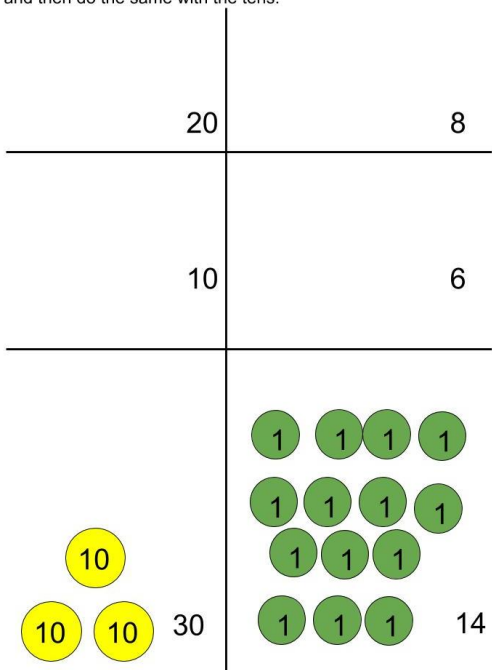
The process is very similar for numbers that bridge 10:

28+16=

28+16=

Use place value knowledge to get the right number of place value counters for each number.

Partition the numbers into tens and units

20	8
10	6

Move the units together and count how many there are and then do the same with the tens.

20	8
10	6

then add each part (units first)

$$\begin{array}{cc} 20 & 8 \\ +\ 10 & 6 \\ \hline 30 & 14 \end{array}$$

Finally we exchange ten units for a ten counter

40

4
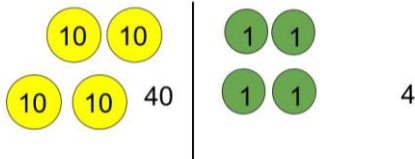
finally, recombine

$$\begin{array}{r} 30 \\ 14 \\ \hline 44 \end{array}$$

This step may need to be explained as, '0 units add 4 units is 4 units.' and ,3 tens add 1 ten is 4 tens.'

Note that for all addition we start with the least significant digit

Stage D: Expanded column method (Not all pupils will need this stage)

Stage of development:

Year Three
- Add numbers with up to three digits

Year Four
- Add numbers with up to 4 digits

Year Five and Six
- Calculations with more than 4 digits including decimals

Underlying skills:
- Understanding the value of each digit in a number
- Being able to add multiples of 10
- Instant recall of addition facts for numbers up to 10
- Know that the hundreds, tens and units should line up appropriately underneath each other (link with place value)

Partitioning and adding the least significant digit first:

2-digit+2-digit

```
  83
  42
 ____
   5      Add the units and record here
 120      Add the tens and record here
 ____
 125      Recombine and record here
```

3-digit +2-digit

```
 159
  26
 ____
  15      Add the units and record here
  70      Add the tens and record here
 100      Add the hundreds and record here
 ____
 185      Recombine and record here
```

Stage E Column method

Stage of development:

Year Three
- Add numbers with up to three digits

Year Four
- Add numbers with up to 4 digits

Year Five and Six
- Calculations with more than 4 digits including decimals

Underlying skills:
- Understanding the value of each digit in a number
- Being able to add multiples of 10
- Instant recall of addition facts for numbers up to 10
- Know that the hundreds, tens, units and decimal points should line up appropriately underneath each other
- Understand the place value in decimals

Partitioning and adding the least significant digit first:

2-digit+2-digit without carrying

33
42
75

Add the **units** and record here

Add the **tens** and record here in the tens column. This can be explained as '3 tens add 4 tens is 7 tens'

With carrying

1
38
46
84

Add the **units** and record here. Because our answer is greater than 9 we only record the units value here (in this case 4 because 8+6=14) and we carry the tens value here

Add the **tens** and record here in the tens column adding in the ten left over from adding the units. This can be explained as '3 tens add 4 tens add one ten is 8 tens'

3-digit + 2-digit without carrying

```
  153
   26
 ─────
  179
```

Add the **units** and record here in the units column

Add the **tens** and record here in the tens column

Add the **hundreds** and record here in the hundreds column

With carrying

```
 1 1
 168
  56
─────
 224
```

Add the **units** and record here in the units column. If the sum of the units is greater than 9 only the unit of the total is recorded the tens is carried here.

Add the **tens** and record here in the tens column remembering to include the ten we have carried. Because the total is greater than 90 only the tens value is recorded in the tens column. The hundreds value is carried here.

Add the **hundreds** remembering to include the hundred we carried earlier and record here in the hundreds column